Prime Number Distribution for 7 and above and a Structure for Goldbach's Strong Conjecture

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Abstract

A simple observation about the most common distance greater than 2 between low prime numbers, leads to a slight unravelling of prime numbers into 8 channels arranged into a palindromic structure which when inverted against itself provides a deep understanding about the variability of pairs of prime numbers that combine to form even numbers. The generation of parametric equations that govern all the non-prime numbers whose factors exclude 3 and 5, reveals that the types of numbers that can be factors for each channel is restricted to discrete combinations. The equations also reveal how these non-prime numbers are paired.

Showing how numbers that are non-divisible by 3 and 5 are distributed leads to the conclusion that prime numbers are evenly distributed throughout these channels with 2 of them varying slightly which supports the relevance of the palindromic structure.

Acknowledgements

Many thanks to Garry Tee of the Department of Mathematics, University of Auckland, New Zealand for providing an overview of Dirichlet's work and that of other authors in this area of research.

Goldbach's Strong Conjecture

This states that all even integers are formed by the addition of at least one pair of prime numbers.

One only has to look at the plots of numbers of pairs of primes that combine to form different even integers over large ranges of numbers to appreciate that something is happening that is causing this phenomenon. It would appear that it should be simple to show some basis for what is obvious from an observational perspective.

Like many problems associated with Number Theory, it should not be necessary to look at very high numbers to try and understand them.

Yet, a simple pattern in pairs of numbers does exist that reveals something quite extraordinary in the sense that it was not found hundreds of years ago. The work of Dirichlet showed that there is a deceasing number of prime numbers (asymptotic) in starting with any number and adding multiples of a second number that has no factors in common with the first number.

Dirchlet's Work

Denote the number of primes <= x by pi(x). In 1896, Hadamard and de la Vallee Poisson proved the (very deep) Prime Number Theorem that pi(x) is asymptotically x/(log x), i.e. as x increases the expression pi(x) / [x/(log x)] converges to 1.

In 1837, Gustav Peter Lejeune Dirichlet proved the deep and very important theorem that, for co-prime positive integers a and d (i.e. gcd(a,d) = 1), the infinite sequence a, a+d, a+2d, a+3d, a+4d, . . . has infinitely many primes. Moreover, the sum of the reciprocals of those primes is divergent.

Later mathematicians refined this in terms of Euler's totient function phi(d), which is defined as the number of integers in [1, 2, 3, ..., d-1] which are co-prime with d. Then if z is ANY one of those phi(d) numbers less than d, as x increases the number of primes in the sequence (a, a+d, a+2d, ...) which are less than x and have the form de+z is asymptotically [x/(log x)] / phi(d). This is called the "Prime Number Theorem for Arithmetic Progressions". [cf. Richard Crandall & Carl Pomerance, Prime Numbers: a Computational Perspective, Springer, New York, 2001, pp.11-12.]

For example, with d=3 there are 2 numbers (1 & 2) less than 3 which are co-prime to 3. Hence as x increases the number of primes <=x which have the form 3e+1 is asymptotically [x/(log x)] / 2; and likewise for primes of the form 3e+2. With d=4, the numbers less than 4 which are co-prime with 4 are 1 and 3. Hence as x increases the number of primes <=x which have the form 4e+1 is asymptotically [x/(log x)] / 2; and likewise for primes of the form 4e+3. With d=9, the numbers less than 9 which are co-prime with 9 are 1,2,4,5,7,8. Hence as x increases the number of primes <= x which have the form 9e+1 is asymptotically [x/(log x)] / 6; and likewise for primes of each of the forms 9e+2, 9e+4, 9e+5, 9e+7, 9e+8.

Common Differences between Prime Numbers

If one were to draw a list of pairs of primes that combined to form the first 100 or so even numbers, what you find is what appears to be random pairings of different numbers. Take any pair of prime numbers and look at the differences between them and higher pairs and it is obvious that the most common difference between prime numbers is a multiple of two.

But there is a more interesting number that is the second most common difference between prime numbers and that is a multiple of 30. This is the product of 2 x 3 x 5.

In order to see the effect of this, you need to put into a table, odd numbers arranged in blocks of 30.

1	**3**	**5**	7	**9**	11	13	**15**	17	19	**21**	23	**25**	**27**	29
31	**33**	**35**	37	**39**	41	43	**45**	47	49	**51**	53	**55**	**57**	59
61	**63**	**65**	67	**69**	71	73	**75**	77	79	**81**	83	**85**	**87**	89

The columns in bold are the columns into which all multiples of 3 and 5 fall into. What remains are eight channels into which all remaining prime numbers must fall. In each channel, prime numbers that fall into that channel are a multiple of 30 above another lower prime number. The first row is the exception. Therefore all prime numbers above 30 are a multiple of 30 above another prime number.

This table shows that multiples of 3 and 5 form a pattern of repetition where the placement of multiples repeats indefinitely in consecutive blocks of 30.

Any number that is not a multiple of 3 or 5 falls into one of these channels also and we can create a multiplication table based on the interaction between numbers that are in these channels to see what distribution patterns exist. This table is known as a Cayley table and it is Abelian which means it is symmetrical.

I have called the remaining channels, the **Prime Number Channels**.

Because all these channels represent a number of 30s + either 1 (or 30Mod1), 7 (or 30Mod7), 11 or

(30Mod11), 13 (or 30Mod13), 17 (or 30Mod17), 19 (or 30Mod19), 23 (or 30Mod23) or 29 (or (30Mod29), we can derive a series of simple parametric equations that describe all the non-prime numbers that fall into these spaces.

For the sake of convenience and understanding I have called these channels Channel 1, Channel 7, Channel 11, Channel 13, Channel 17, Channel 19, Channel 23 and Channel 29.

For example $(30A + 1) \times (30B + 7) = 900AB + 30B + 210A + 7$ is a number that is in channel 7.

The arrangement of these channels is a palindromic mirror image of itself. From both left and right, the Prime Number Channels are 1, 7 and 11 and 13 positions away from the nearest multiple of 30.

Now look what happens as a second layer of numbers slides across the top layer to the left, one at a time to see what alignments (where top and bottom numbers are not bold) occur for each line.

Table 1 - Possible Alignments of Prime Number Channels

Relative Positions	Aligned Channels	Alignments
1 **3** **5** 7 **9** 11 13 **15** 17 19 **21** 23 **25** **27** 29 **33** **35** 37 **39** 41 43 **45** 47 49 **51** 53 **55** **57** 59 61	11+43,17+49,29+61	3
1 **3** **5** 7 **9** 11 13 **15** 17 19 **21** 23 **25** **27** 29 **35** 37 **39** 41 43 **45** 47 49 **51** 53 **55** **57** 59 61 **63**	7+41,13+47,19+53	3
1 **3** **5** 7 **9** 11 13 **15** 17 19 **21** 23 **25** **27** 29 37 **39** 41 43 **45** 47 49 **51** 53 **55** **57** 59 61 **63** **65**	1+37,7+43,11+47 13+49,17+53,23+59	6
1 **3** **5** 7 **9** 11 13 **15** 17 19 **21** 23 **25** **27** 29 **39** 41 43 **45** 47 49 **51** 53 **55** **57** 59 61 **63** **65** 67	11+49,23+61,29+67	3
1 **3** **5** 7 **9** 11 13 **15** 17 19 **21** 23 **25** **27** 29 41 43 **45** 47 49 **51** 53 **55** **57** 59 61 **63** **65** 67 **69**	1+41,7+47,13+53 19+59	4
1 **3** **5** 7 **9** 11 13 **15** 17 19 **21** 23 **25** **27** 29 43 **45** 47 49 **51** 53 **55** **57** 59 61 **63** **65** 67 **69** 71	1+43,7+49,11+53,17+59 19+61,29+71	6
1 **3** **5** 7 **9** 11 13 **15** 17 19 **21** 23 **25** **27** 29 **45** 47 49 **51** 53 **55** **57** 59 61 **63** **65** 67 **69** 71 73	17+61,23+67,29+73	3
1 **3** **5** 7 **9** 11 13 **15** 17 19 **21** 23 **25** **27** 29 47 49 **51** 53 **55** **57** 59 61 **63** **65** 67 **69** 71 73 **75**	1+47,7+53,13+59	3
1 **3** **5** 7 **9** 11 13 **15** 17 19 **21** 23 **25** **27** 29 49 **51** 53 **55** **57** 59 61 **63** **65** 67 **69** 71 73 **75** 77	1+49,11+59,13+61,19+67 23+71,29+77	6
1 **3** **5** 7 **9** 11 13 **15** 17 19 **21** 23 **25** **27** 29 **51** 53 **55** **57** 59 61 **63** **65** 67 **69** 71 73 **75** 77 79	11+61,17+67,23+73,29+79	4
1 **3** **5** 7 **9** 11 13 **15** 17 19 **21** 23 **25** **27** 29 53 **55** **57** 59 61 **63** **65** 67 **69** 71 73 **75** 77 79 **81**	1+53,7+59,19+71	3
1 **3** **5** 7 **9** 11 13 **15** 17 19 **21** 23 **25** **27** 29 **55** **57** 59 61 **63** **65** 67 **69** 71 73 **75** 77 79 **81** 83	7+61,13+67,17+71,19+73 23+7,29+83	6
1 **3** **5** 7 **9** 11 13 **15** 17 19 **21** 23 **25** **27** 29 **57** 59 61 **63** **65** 67 **69** 71 73 **75** 77 79 **81** 83 **85**	11+67,17+73,23+79	3
1 **3** **5** 7 **9** 11 13 **15** 17 19 **21** 23 **25** **27** 29 59 61 **63** **65** 67 **69** 71 73 **75** 77 79 **81** 83 **85** **87**	1+59, 13+71,19+77	3
1 **3** **5** 7 **9** 11 13 **15** 17 19 **21** 23 **25** **27** 29 61 **63** **65** 67 **69** 71 73 **75** 77 79 **81** 83 **85** **87** 89	1+61,7+67,11+71,13+73 17+77,19+79,23+83,29+89	8

In each alignment of blocks of 30, at least three channels are aligned. The relative arrangements of channels produces the highest number of Prime Number Channel alignments when the individual channels are aligned with themselves. Minor alignments are when the multiples of 3 align with multiples of 5 maximally, followed by the alignments of multiples of 5 (e.g. 1 and 51) and the remainder where three Prime Number Channels align.

This shows that no matter how the channels are aligned, there are at least three alignments of Prime Number Channels. The mirror-image symmetrical structure of a block of 30 also means that when the second layer is completely reversed left-to-right, a similar set of alignments occur for what is now a representation of what happens with Goldbach's Strong Conjecture. This clearly demonstrates that in the worst-case scenarios, there are still three pairs of channels that align in each block of 30. Also, it is impossible for more than one channel to align with itself. The result of that is because the distribution of prime numbers in each channel is different, when channels are aligned and inverted against either themselves or other channels, they are most unlikely to have all non-prime number positions align perfectly.

Thirty and its multiples are the most common differences between prime numbers other than two. Other differences are a multiple of 30 plus a number between 2 and 28 inclusive.

So for each block of 30, depending on how the two lines numbers align up different channels, the probability that a number of prime numbers pairs exists in a particular alignment is controlled by the number of alignments.

The following tables show the differences between Prime Number Channels that are less than 30 and the frequency of each difference.

Table 3 – Differences between low PNC Numbers

	1	**7**	**11**	**13**	**17**	**19**	**23**	**29**
7	6							
11	10	4						
13	12	6	2					
17	16	10	6	4				
19	18	12	8	6	2			
23	22	16	12	10	6	4		
29	28	22	18	16	12	10	6	
31		24	20	18	16	12	8	2
37			26	24	20	18	14	8
41				28	24	22	18	12
43					26	24	20	14
47						28	24	18
49							26	20
53								24

Difference	**Frequency**
2	3
4	3
6	6
8	3
10	4
12	6
14	3
16	3
18	6
20	4
22	3
24	6
26	3
28	3

The frequency of differences between prime numbers combined with the almost even spread of prime numbers between the Prime Number Channels offers a tantalisingly close feeling for understanding Goldbach's Conjecture at an even deeper level.

As expected, multiples of 6 are the next most common difference, followed by multiples of 5.

The following multiplication table shows individual channel numbers as the factors with the results in the body of the table. If two numbers are multiplied by each other and they are a 30Mod1 number and a

30Mod7 number, the result will be a 30Mod7 number

i.e. (30A x 30B) + (7 x 30A) +(1 x 30B) + (1 x 7).

The following table has been abbreviated to show the effect of multiplying the different Modulo numbers and has been shown in a simple format. The top row and left-most column are the Modulo factors and the intersecting values are the resulting Modulo products.

Table 4 – Channel Multiplication Products

	1	**7**	**11**	**13**	**17**	**19**	**23**	**29**
1	1	7	11	13	17	19	23	29
7		19	17	1	29	13	11	23
11			1	23	7	29	13	19
13				19	11	7	29	17
17					19	23	1	13
19						1	17	11
23							19	7
29								1

The first row of the table shows that each channel is represented once. To read off the other numbers, simply start beneath the column title and go down until you get to the last number, then continue to the right. For example, under the heading 7, its Modulo products follow 7 and 19, and then reading to the right, 17, 1, 29, 13, 11 and 23.

The remaining numbers that are multiples of 3 and 5 obviously land on multiples of 3 and 5 and so each number multiplied by the successive numbers in a block of 15 odd numbers, lands on just one channel that is not repeated by any of the following numbers in that same block.

Therefore non-prime numbers are evenly spread throughout the prime number channels as well as the full range of numbers in a block of 30.

There are a total of 36 possible combinations of channel multiplication and we can see how these are split if they are grouped together by the resultant channel or Modulo product.

Table 5 – Paired 30ModX Factors for each Prime Number Channel

Channel	Factor 1	Factor 2
1	1	1
	7	13
	11	11
	17	23
	19	19
	29	29
7	1	7
	11	17
	13	19
	23	29
11	1	11
	7	23
	13	17
	19	29
13	1	13
	7	19
	11	23
	17	29
17	7	11
	1	17
	13	29
	19	23
19	7	7
	13	13
	17	17
	1	19
	23	23
	11	29
23	11	13
	17	19
	1	23
	7	29
29	7	17
	11	19
	13	23
	1	29

There are two immediately obvious things in this table. Firstly Channels 1 and 19 have 6 sets of factors whereas all others have 4. Secondly, 7 and 17, 13 and 23, 1 and 11, 19 and 29 behave quite differently in multiplication.

Up to the square root of a number to test as a prime, numbers in each of the eight prime number channels still need to be tested as potential factors.

Briefly returning to the observation that non-prime numbers must be evenly spread throughout the prime number channels we can now reasonably and safely assume that prime numbers must be relatively uniform across the 8 prime number channels.

A simple computer program that displays on screen in real time, the number of prime numbers in each Prime Number Channel as 8 numbers to be tested are incremented by 30, shows a pattern that when averaged out over large ranges of numbers shows remarkable consistency in numbers between the different channels. Different channels seem to spring into life and then remain dormant while other ones become active. And two channels stand out as having about 0.5% to 0.7% less prime numbers and these are Channel 1 and Channel 19.

There is at least one apparent reason for this. If you look back at the multiplication table, you will see that channels multiplied by themselves produce only Channel 1 or Channel 19 results. This is material because the first time a non-prime number derivative is seen for a prime number is when it is a square of itself. When multiplied by a lower number (i.e. 7 x 3), it belongs to that lower number's (3) downstream series of multiples. Therefore in any numerical range, there are going to be some numbers that are seen as prime numbers and only as squares before the range increases to include their other downstream multiples greater than squares. It also means that when a prime number (P) is seen, its first non-prime derivative is the square of itself or (P-1)P higher from P.

Understanding the channel multiplications provides a way of creating 36 parametric equations describing all non-prime numbers that are not multiples of 3 or of 5.

Channel 7 Equations (30Mod7)
From the list of factors for channel 7 above, we have 1 and 7, 11 and 17, 13 and 19, and 23 and 29.

A) 1 and 7 => $(30A+1)(30B+7) = (900AB + 210A + 30B) + 7$
B) 11 and 17 => $(30A+11)(30B+17) = (900AB + 510A + 330B) + 187$

Look at each set of factors for each channel and you can see that there are pairs of numbers that are equidistant from each 30 boundary and in the next two equations this can be used to demonstrate the behaviour of all non-prime numbers above a certain number of being equidistant from other numbers.

C) 13 and 19 are treated as -17 and -11

=> $(30A-17)(30B-11) = (900AB - 510A - 330B) + 187$

D) Similarly 23 and 29 are -7 and -1

=> $(30A-7)(30B-10 = (900AB - 210A - 30B) + 7$

Compare A) and D). Both are equidistant from 900AB by (210A + 30B) and (-210A - 30B) with an offset of 7 added to each one.

Compare B) and C). Both are equidistant from 900AB by (510A + 330B) and (-510A - 330B) with 187 added.

Equations for Channel 1.

A) (900AB + 30A + 30B) + 1
B) (900AB - 30A - 30B) + 1
C) (900AB + 330A + 330B) + 121
D) (900AB - 330A - 330B) + 121
E) (900AB + 390A + 210B) + 91
F) (900AB - 390A - 210B) + 91

Equations for Channel 19

A) (900AB + 210A + 210B) + 49
B) (900AB – 210A – 210B) + 49
C) (900AB +390A + 390B) + 169
D) (900AB – 390A – 390B) + 169
E) (900AB + 570A + 30B) + 19
F) (900AB – 570A – 30B) + 19

Equations for Channel 11

A) (900AB + 330A + 30B) + 11
B) (900AB – 330A – 30B) + 11
C) (900AB + 690A + 210B) + 161
D) (900AB – 690A – 210B) + 161

Equations for Channel 13

A) (900AB + 390A + 30B) + 13
B) (900AB – 390A – 30B) + 13
C) (900AB + 570A + 210B) + 133
D) (900AB – 570A – 210B) + 133

Equations for Channel 17

A) (900AB + 330A + 210B) + 77
B) (900AB – 330A – 210B) + 77
C) (900AB + 510A + 30B) + 17
D) (900AB - 510A - 30B) + 17

Equations for Channel 23

A) (900AB +390A + 330B) +143
B) (900AB – 390A -330B) + 143
C) (900AB + 570A + 510B) + 323
D) (900AB - 570A – 510B) +323

Equations for Channel 29

A) (900AB + 510A + 210B) + 119
B) (900AB – 510A – 210B) + 119
C) (900AB + 570A + 330B) + 209
D) (900AB – 570A – 330B) + 209

In the equations for Channel 19 above, 4 equations have identical constants used to multiply A and B, while the last two do not. When factorising AB therefore, E) and F) produce twice as many non-prime numbers as the first four. Therefore for any unique A and B, Channel 1 and Channel 19 equations A)->D) produce 4 non-prime numbers while E) and F) respectively produce 4 which is a total of 8.

Channels 7, 11, 13, 17, 23 and 29 all have four equations that produce two non-prime numbers each for each unique A and B value and because there are four of them, they also produce a total of 8 non-prime numbers per channel. Overall each channel produces 8 non-prime numbers per unique A and B and therefore it is reasonable to say that the explanation above for why Channels 1 and 19 have fewer prime numbers is probably right or very close to the mark.

Each channel's multiplication products is also the mirror image of another pair. The following table shows the sequence of channels in which multiplication products fall into and places it alongside the other member's products of that pair for comparison. Note that the members of each pair are the same distance from a multiple of 30, one positive and one negative.

Channel 6: Multiplication Channel Pairs

Channel	**Multiplication Channel Sequence**							
1	1	7	11	13	17	19	23	29
29	29	23	19	17	13	11	7	1
7	7	19	17	1	29	13	11	23
23	23	11	13	29	1	17	19	7
11	11	17	1	23	7	29	13	19
19	19	13	29	7	23	1	17	11
13	13	1	23	19	11	7	29	17
17	17	29	7	11	19	23	1	13

Each channel's complementary pair has the exact opposite sequence of channels that its multiplication products fall into. For Channel 1, the channels can be completely reversed to get the channel sequence that Channel 29 products fall into. Because the sequence of the members of each pair are inverted, it also leads to the sums of the sequence channels adding up to 30.

This symmetry also extends to the way that each channel's multiplication sequence aligns with the other channels to produce 30.

Table 7 – Vertical and Horizontal Symmetry

1	7	11	13	17	19	23	29
7	**19**	**17**	**1**	**29**	**13**	**11**	23
11	**17**	1	23	7	29	**13**	19
13	**1**	23	**19**	**11**	7	**29**	17
17	**29**	7	**11**	**19**	23	**1**	13
19	**13**	29	7	23	1	**17**	11
23	**11**	**13**	**29**	**1**	**17**	**19**	7
29	23	19	17	13	11	7	1

The above table shows the multiplication channel sequences for each channel. Each pair of numbers that are equidistant from their respective sides (traveling vertically or horizontally), adds up to 30. The first line and the first column must follow this pattern (1+29, 7+23, 11+19,13+17) but it is also followed by the remaining pairs of numbers that are equidistant from multiple of 30 boundaries. The table is also a set of nested rectangles and this is shown by the alternating bold and underlined characters. Use the lines of each rectangle to observe the pairing of numbers that are at the other end of the rectangle.

In summary, there is similarity between each of the Prime Number Channels. Each channel produces the same number of non-prime numbers and is characterised by pairs of equations that act in tandem.

Notice also that the differences for Channels 1 and 19 between the first two equation offsets and the next two equation offsets is 120 – another similarity.

Each Prime Number Channel has a set of parametric equations with the same characteristic of paired equations. It appears clear that the distributions of prime numbers within each channel are very dissimilar due to the different constants used in each equation in each channel.

Removing Multiples of 3

If multiples of three are ignored, the remaining prime numbers fall into two groups, described by the equations 6N+1 and 6N-1. With three included, all prime numbers are described by the traditional equations 2N+1 and 2N-1.

The behaviour of these 6N+1 and 6N-1 is worth inspecting more closely for it shows how the squares of numbers behave from this perspective. A and B are used in the following table to allow for different factors.

Table 8: 6N+1 and 6N-1 Multiplication

	6A+1	**6B-1**
6C+1	6N+1	6N-1
6D-1	6N-1	6N+1

Over6 numbers (6N+1) and Under6 numbers (6N-1) always produce Over6 numbers through a simple expansion of (6A+1)(6B+1) to 36AB + 6A + 6B + 1 and (6A-1)(6B-1) to 36AB - 6B – 6A + 1.

It is clear that the different equations for each Prime Number Channel results in different distributions of non-prime numbers and therefore the distribution of prime numbers will be different in each channel. This is influential in understanding the pairing of prime numbers and why the alignment of different channels is likely to form form pairings.

Without looking at the subsequent Prime Number Definitions given later, it may be possible to formulate a proof for Goldbach's Strong Conjecture by combining Gauss's observation about the density of prime numbers with the Prime Number Channels and the nature of their alignments at different positions. There may also be benefit looking at the observation that the differences between prime numbers includes the complete set of even numbers.

Understanding of the Prime Number Channels provides a framework for gaining an understanding of Goldbach's Strong Conjecture, not only for the alignment of different channels but also for the differences in matches per alignment.

It is clear that as the product AB increments, that the other components (e.g. 30A + 30B) start to extend beyond the range of 900. The other observation is that the factors of the product AB provide for situations where although the product AB is the same, 30A and 30B do change.

For example - A=10 and B=2
(900 x 20) + (30 x 10) + (30 x 2) + 1 = 2161 doesn't intersect with
(900 x 20) + (30 x 5) + (30 x 4) + 1 = 2071.

Therefore when AB contains many factors, the different products fill many positions. But interestingly when AB is a prime number then it doesn't fill more than one position suggesting that larger prime numbers exist because smaller prime numbers exist.

Taking just one of the equations gives an understanding of why it produces prime number positions relative to that equation. Take the equation for 900AB + 30A + 30B + 1 from above.

For any value of AB, the prime number positions relative to it are sets of 30 multiplied by the remaining numbers less than AB that also exclude the sums of all the possible factors of AB. The following table shows the non-prime number positions filled in a short range.

Table 9: Non-Prime Numbers for Channel 1 – (900AB + 30A + 30B)+1 Equation

900AB	AB	A	B	30A	30B	30A+30B	900AB+30A+30B+1
900	1	1	1	30	30	60	961
1800	2	1	2	30	60	90	1891
2700	3	1	3	30	90	120	2821
3600	4	1	4	30	120	150	3751
		2	2	60	60	120	3721
4500	5	1	5	30	150	180	4681
5400	6	1	6	30	180	210	5611
		2	3	60	90	150	5551
6300	7	1	7	30	210	240	6541
7200	8	1	8	30	240	270	7471
		2	4	60	120	180	7381
8100	9	1	9	30	270	300	8401
		3	3	90	90	180	8281
9000	10	1	10	30	300	330	9331
		2	5	60	150	210	9211

In each set of 900, the relative prime number positions are all the positions not marked by AB or the sum of 30A+30B, but multiplied by 30. For example 8101 has relative prime number positions at distances of 30 x 1, 2, 3, 4, 5, 7 and 8, but also 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26,27,28 and 29 which are up to the next block of 900.

When looking at the other pair of this equation. It is possible to see how it largely fails to intersect with the first equation as it starts at the 900 boundary and works back the other way.

All Prime Number Channel equations produce pairs of non-prime numbers centred around a common point and that raises the prospect that "raw" (in respect to that common point and not taking into consideration the effects of paired non-prime numbers around other points) prime numbers also exist in pairs centred around the same point.

Suggestion for Further Research

1-Take one of the channels and select one of the equations for that channel. See if there a way of showing how certain positions are avoided by the various values of the parameters A and B. If someone could do this, then they would then be able to do the same for the other individual equations for that channel and then find situations where each equation missed the same point.

For example are there variations of an equation that increases the probability of retrieving prime numbers? Take the equation for channel 7 –> 900AB +210A + 30B + 7.

Variations of these could include;

900AB + 420A + 30B + 7
900AB + 210A + 60B + 7
900AB + 210A + 210B + 7
900AB + 30A + 30B + 7

Ignoring the offset of 7, if any of the above sets of numbers do not have 7 as a factor, then it is not divisible by 7.

When A=1 and B=2, the last equation variation gives 900 x 2 + 90 + 7.
900 x 2 + 90 = (20 x 90) + 90 = 21 x 90. which is divisible by 7.

Say for example if the number of 90's was a multiple of 37, 67 or other 30Mod7 number before the offset of 7 were added, that would guarantee that the result was not divisible by 7 or that 30Mod7 number.

It's a simple matter to show that one equation cannot fill all positions in the channel it populates with non-prime numbers. Take Channel 7 for instance. The equation (900AB + 210A + 30B) + 1 completely avoids positions populated by the other equations where for each of these other equations, the factors are exclusively those found for those equations. For example, (30A+11)(30B+17) where (30A+11) has factors that are exclusively (30X+11) numbers and (30B+17) has factors that are exclusively (30Y+17) numbers. This therefore applies to each equation in each channel.

2-Construct a table that looks at the numbers of pairs of primes for a consecutive block of numbers. For example, to test the number of pairs for the ranges of consecutive even integers 60->118, 600->658, 6000->6058, 60000->60058 … you would use the base numbers from 1->59, 1->329, 1-3029, 1-30029 … and invert them against themselves, find the number of pairs of primes that sum together to produce the first number in each consecutive series and then displace the inverted list by 1 to the right and repeat this process until the end of the block. The purpose of this is to establish if the relative percentages of pairs of primes per alignment position in a a consecutive block of numbers mirrors what happens in the first few blocks.

3-Look at each equation for each prime number channel and verify the number of prime number positions relative to each equation for its channel for large ranges of numbers. Some of these equations may produce more non-prime number positions than others although there will be balancing forces. For example Channel 1 and 19 have 6 equations each but the relative number of primes in each of these channels is only slightly less than the others.

4-Explore how different equations in each channel fill positions relative to each other.

5- Analyse the relative number of Prime Numbers appearing in real time in each channel to see if there is a pattern. This could be for example to find out whether some channels tend to generate more prime numbers predominantly before being caught up by other channels. The pairing of channels with inverted multiplication channel sequences and the fact that not all possible permutations of channel sequences are used indicates that patterns of multiplication have a part to play in prime number positions.

6-Analyse the individual channels to find out which equations are the dominant providers of non-prime numbers.

7-Analyse the Non-Prime numbers produced by an individual channel equation by looking at the multiples of 30 that divide into it. For example Channel 7, equation 900AB + 210A + 30B + 1. Deduct 1 from a range of results and look for patterns in the factors and numbers.

Following is a table that shows some of these results and there is an interesting pattern that appears here to do with Prime Numbers. A and B are various factors of A and B, while x30 is the number of 30's in 900AB, 7A is the number of 30's in 210A, B is the number of 30's in 30B, Total is the number of 30's in the total, Prime1 shows if Total is a prime number, Prime2 shows if 7A+B is a prime number, A+B is self-explanatory and Prime 3 shows if A+B is a prime.

Table 10: Channel 7 - (30A+1)(30B+7) Equation – 30Mod1 Analysis

A	B	x30	7A	B	Total	Prime1	7A+B	Prime2	A+B	Prime3
1	1	30	7	1	38		8		2	
2	1	60	14	1	75		15		3	yes
1	2	60	7	2	69		9		3	yes
3	1	90	21	1	112		22		4	
1	3	90	7	3	100		10		4	
4	1	120	28	1	149	yes	29	yes	5	yes
1	4	120	7	4	131	yes	11	yes	5	yes
2	2	120	14	2	136		16		4	
5	1	150	35	1	186		36		6	
1	5	150	7	5	162		12		6	
6	1	180	42	1	223	yes	43	yes	7	yes
1	6	180	7	6	193	yes	13	yes	7	yes
3	2	180	21	2	203		23	yes	7	yes
2	3	180	14	3	197	yes	17	yes	7	yes
7	1	210	49	1	260		50		8	
1	7	210	7	7	224		14		8	
8	1	240	56	1	297		57		9	
1	8	240	7	8	255		15		9	
4	2	240	28	2	270		30		6	
2	4	240	14	4	258		18		6	
9	1	270	63	1	334		64		10	
1	9	270	1	9	280		10		10	
3	3	270	21	1	292		22		6	
10	1	300	70	1	371		71	yes	11	yes
1	10	300	7	10	317	yes	17	yes	11	yes
5	2	300	35	2	337	yes	37	yes	7	yes
2	5	300	14	5	319		19	yes	7	yes
11	1	330	77	1	408		78		12	
1	11	330	7	11	348		18		12	
12	1	360	84	1	335		85		13	yes
1	12	360	7	12	379	yes	19	yes	13	yes

6	2	360	42	2	404		44		8	
2	6	360	14	6	380		20		8	
4	3	360	28	3	391		31	yes	7	yes
3	4	360	21	4	385		25		7	yes
13	1	390	91	1	482		92		14	
1	13	390	7	13	410		20		14	
14	1	420	98	7	525		105		15	
1	14	420	7	14	441		21		15	
7	2	420	49	2	471		51		9	
2	7	420	14	7	441		21		9	
15	1	450	105	1	556		106		16	
1	15	450	7	15	472		22		16	
5	3	450	35	3	488		38		8	
3	5	450	21	5	476		26		8	
16	1	480	112	1	593	yes	113	yes	17	yes
1	16	480	7	16	503	yes	23	yes	17	yes

When Prime1 is a prime number, it appears that Prime 2 and Prime 3 are also prime numbers in this limited range. More in-depth analysis of this equation and others may provide the reasons why prime numbers appear grouped in this way.

The following table uses equation 2 - (900AB+510A + 330B) + 187 and this shows that something similar is happening here and probably in all equations. This works for this equation if the offset of 187 is left aside and not used to add in another potential 6 lots of 30.

Table 11: Channel 7 - (30A+11)(30B+17) Equation – 30Mod1 Analysis

A	B	x30	17A	11B	Total	Prime1	17A+11B	Prime2	A+B	Prime3
1	1	30	17	11	58		28		2	
2	1	60	34	11	105		45		3	yes
1	2	60	17	22	99		39		3	yes
3	1	90	51	11	152		62		4	
1	3	90	17	33	140		50		4	
4	1	120	68	11	199	yes	79	yes	5	yes
1	4	120	17	44	181	yes	61	yes	5	yes
2	2	120	34	22	176		56		4	
5	1	150	85	11	246		96		6	
1	5	150	17	55	222		72		6	
6	1	180	102	11	293		113		7	yes
1	6	180	17	66	263		83		7	yes
3	2	180	51	22	253		73	yes	5	yes
2	3	180	34	33	247		67	yes	5	yes
7	1	210	119	11	340		130		8	
1	7	210	17	77	304		94		8	
8	1	240	136	11	387		147		9	
1	8	240	17	88	345		105		9	
4	2	240	68	22	330		90		6	

2	4	240	34	44	318		78		6	
9	1	270	153	11	434		164		10	
1	9	270	17	99	386		116		10	
3	3	270	51	33	354		84		6	
10	1	300	170	11	481		181	yes	11	yes
1	10	300	17	110	427		127	yes	11	yes
5	2	300	85	22	407		107	yes	7	yes
2	5	300	34	55	389	yes	89	yes	7	yes
11	1	330	187	11	528		198		12	
1	11	330	17	121	468		138		12	
12	1	360	204	11	575		215		13	yes
1	12	360	17	132	509		149	yes	13	yes
6	2	360	102	22	84		124		8	
2	6	360	34	66	460		100		8	
4	3	360	68	33	461	yes	101	yes	7	yes
3	4	360	51	44	455		95		7	yes

Two initial observations appear to ensue from the above tables; 1) When there are no primes, there is a common factor for each of the last three numerical columns, and 2) that if the Total is a prime then the other two later columns are also prime. The difference between the Total column and the next numerical column is the x30 column. This is a multiple of 30 and other than 3, this is the next most common difference between prime numbers.

Also, if Prime2 is a prime number, does Prime3 have to be also? When testing greater ranges exceptions occur so it it not a consistent observation.

Phases of Multiplication

In any of the prime number channels, one parametric equation can intersect with another. But there is also a simple mechanism that prevents them intersecting in other situations that is based on sets of 30.

Consider the series 7 x 31, 61, 91, … which occurs in Prime Number Channel 7

Now count the number of blocks of 30 in each set of factors and their product. I will use the term "Phase" to describe where in the set of phases of 0 -> 150 via steps of 30, the factors and their product reside.

Table 12: Phases of Channel 7

Factors	Phase of Factors	Product	Phase of Product
7 x 31	30	217	30
7 x 61	60	427	60
7 x 91	90	637	90
7 x 121	120	847	120
7 x 151	150	1057	150
7 x 181	0 (or 180)	1267	0 (or 180)

The difference between products is 210, or 1 set of phases (180) + 30. You can also see that the difference between 7 x 31 and 7 x 61 is merely 7 x 30.

If the series of factors is used where 31 stays constant and 7 is incremented by 30 successively, the same movement from one phase to the next is observed with a simple transition from 7 x 31 -> 37 x 31 or an increase of 30 x 31 which is the same as (5 x 180) + 30. Therefore any series where one factor increments by steps of 30 will naturally advance a phase at each step before it rolls into a new set of phases.

Suggestion for further research - Are the sets of phases actually larger than 180? i.e. are they multiples of 180?
Find examples of different series that land on the same location.
The simplest example of two numbers that land on the same location is 7 x 7 x 7 x 7 x 7 x 31 (NB: a multiple of 30) + 7) ^ 5 falls into same phase again as)
16807 x 31 = 7 x (2401 x 31) (using the same 7, 37,... x 31, 61, ... series)
and also (7 x 7) x (343 x 31) in the 19, 49, ... x 13, 43, ... series)
and also (7 x 7 x 7) x (49 x 31)
and (7 x 7 x 7 x 7) x (7 x 31)

Other Sets of Factors of Channel 7
There are three other sets of factors that populate Channel 7 with non-prime numbers and each of them has sets of phases although they start at different points and go up or down.

Table 13 – Phases for 30Mod17 by 30Mod11 Multiplication

Factors	Phase of Factors	Product	Phase of Product
17 x 41	30	697	150
17 x 71	60	1207	120
17 x 101	90	1717	90
17 x 131	120	2227	60
17 x 161	150	2737	30
17 x 191	0 (or 180)	3247	0 (or 180)

The next two sets of factors have product phases offset from the sum of 30's of the respective factors. Again, one series of product phases increases while the second one decreases.

Table 14 – Phases for 30Mod13 by 30Mod19 Multiplication

Factors	Phase of Factors	Product	Phase of Product
13 x 19	0	247	60
13 x 49	30	637	90
13 x 79	60	1027	120
13 x 109	90	1417	150
13 x 139	120	1807	0 (or 180)
23 x 29	0	667	120

23 x 59	30	1357	90
23 x 89	60	2047	60
23 x 119	90	2737	30
23 x 149	120	3427	0 (or 180)
23 x 179	150	4117	150
23 x 209	0	4807	120

We need to check if the alternative series also work this way.

Table 15 – Phases for Prime Number Channel 7 Multiplication

Factors	Phase of Factors	Product	Phase of Product
31 x 7	30	217	30
31 x 37	60	1147	60
31 x 67	90	2077	90
31 x 97	120	3007	120
31 x 127	150	3937	150
11 x 17	0	187	0
11 x 47	30	517	150
11 x 77	60	847	120
11 x 107	90	1177	90
11 x 137	120	1507	60
43 x 19	30	817	90
73 x 19	60	1387	120
103 x 19	90	1957	150
133 x 19	120	2527	0 or (180)
53 x 29	30	1537	90
83 x 29	60	2407	60
113 x 29	90	3277	30
143 x 29	120	4147	0 (or 180)

Although the phases of the factors and products may be different, there is a simple consistent pattern that is irrespective of the size of the factors.

Here is a table of the relationships between the phase of the factors and the phase of the product for these equations. The starting factors are shown in each case and these increment in steps of 30. Once a phase hits 180 it can be treated as 0.

Table 16 – Phase Relationships for Prime Number Channel 7 Multiplication

Factor 1	Factor 2	Factors Phase	Product Phase	Relationship
7	31	A	B	A = B
17	41	A	B	A + B = 180

13	19	A	B	A + 60 = B
23	29	A	B	A + B = 120

Using Phases for Testing of Primeness
The first step in determining how to test if a number is a prime number, is to determine which channel it lies in which is a simple (N mod 30) operation.

The second step is to solve the parametric equations after the offset is deducted from N. This is likely to be time consuming though.

For channel 1 numbers the following equations can be solved.

A)	(900AB + 30A + 30B)	=	N-1
B)	(900AB - 30A - 30B)	=	N+1
C)	(900AB + 330A + 330B)	=	N+121
D)	(900AB - 330A - 330B)	=	N+121
E)	(900AB + 390A + 210B)	=	N+91
F)	(900AB - 390A - 210B)	=	N+91

When solving these equations for a particular channel, the sum of A+B determines if the result will be in phase with the test number. For those equations that have an offset that is not equal to 1, the phase changes and the equation to solve needs to recognise this in order to accomplish this in the fastest way possible.

Here is a sample of various values of A and B applied to Equation A) above.

Table 17 – Phases for different Parameters for Channel 7 Multiplication

A	**B**	**Value of Equation**	**Phase**
1	1	900 + 60	60
2	2	3600 + 120	120
1	2	1800 + 90	90
3	3	8100 + 180	0 (180)
1	3	2700 + 120	120
2	3	5400 + 150	150

For this equation A + B must match the same phase as the result.

There is a simple simplification still available for all these functions and that is to divide all results by 30. This gives the following equations for Channel 1

A)	(30AB + A + B)	=	(N-1)/30
B)	(30AB - A - B)	=	(N+1)/30
C)	(30AB + 11A + 11B)	=	(N+121)/30
D)	(30AB - 11A - 11B)	=	(N+121)/30

E)	(30AB + 13A + 17B)	=	(N+91)/30
F)	(30AB - 13A - 17B)	=	(N+91)/30

The positions of prime numbers in each channel are determined by criteria that are different for each channel. If they had the same equations, prime numbers would land in each channel at the same time as they landed in other channels. That clearly does not happen. The prospects of producing one direct method for finding prime numbers is improbable.

The most important question to have been answered is why any one of the above equations is unable to reach all possible positions for that channel, and at the same time it is only able to reach a lower proportion of those positions. Each equation has its own private set of primes, but some of these will be landed on by other equations for that channel, the net result of cancellation of other equations producing the true primes for that channel.

Discussion of Prime Number Channels

As a prime number, 3 stands out by itself. It never intersects with the prime number channels. The next prime number 5 doesn't either and it is remarkable that they both repeat their pattern within sets of 30.

The higher multiples of the next prime number 7, are the most common non-primes of the prime number channels, repeating the same channel distribution every 7 x 30 blocks.

As successive prime numbers appear, they span greater and greater numbers of blocks of 30 before repeating.

For channel 7, a much faster way of identifying which equations intersect at a particular number is to identify the phase that the number lies in.

Then start with the largest possible factor and smallest possible factor whose product has the same phase as the test number.

Then decrease the largest number, while increasing the smallest number by sets of 30 until they occupy the same phase again. Then either multiply them out and use the difference between their product and the test number to add and subtract additional sets of 30.

Alternatively, it is possible to calculate the effect of moving one factor by several sets of 30, and then moving the other factor by the required number of sets of 30. This provides for smaller calculations compared to multiplying out two test factors. It should also be able to make it easy to make large jumps over zones where one of the potential factors needs to be increased or decreased by larger numbers of sets of 30.

If in each of these channels, this process was followed for one equation at a time, the processing time would most likely be a fraction of doing it the long way with division. Division is simple but doesn't take into account the numbers that are out-of-zone.

This method could either concentrate one at a time on each possible pair of channels whose multiplication product was in the same channel as the number to test, or each pair recently tested could be adjusted to retrieve the next closest pair.

If the alternative product (of two potential factors) were to be recorded, it provides a simple way of adjusting the factors. For example if the number to be tested is C, and A the factor being tested and it multiplies out by say B to be in the next highest block of 30, then A can be increased by A1 to get to the next highest channel candidate, and B decreased by B1 to get to the next lowest channel candidate so that the work required is reduced.

If A x B = C, then (A+A1) x (B-B1) = AB +A1B – B1A

So if the previous total (or C) was recorded along with A and B, the next step would be to multiply out A1B and B1A and compare them against the number tested for primeness.

Combining this approach with channel recording will be useful in reducing the amount of calculations required.

Overall Discussion

Understanding how each of these equations works will lead to a better understanding of prime numbers. The key here is not to focus on producing direct methods for creating prime numbers, but to understand prime numbers as a by-product of a much deeper understanding of non-prime numbers.

Euclid proved that there are an unlimited number of prime numbers but like all others was not able to show how this works. There appears to be nothing that can be used to uniquely identify prime number positions and we are left struggling to explain why these positions continue to appear as numerical ranges increase. Therefore, I am going to offer a short description of why I think there are an infinite number of prime numbers.

There are four basic effects to consider;

1) for every 30 consecutive numbers that a number is multiplied by, 15 of these products fall on even numbers, 7 fall on multiples of 3 and 5, and the remaining 8 fall into the prime number channels.
2) For each new prime number, the expanse over which it's multiplication products span for a consecutive series of 30 numbers is larger than the previous ones.
3) Larger numbers absorb more numbers as factors
4) Many numbers fail to fall into a zone through multiplication.

All these effects combine to form an overall balancing effect that ensures continuing prime numbers being found.

Euclid's proof for an infinite number of prime numbers can be modified in the light of knowledge about the Prime Number Channels. Consider Channel 7, equation 1 - 30(A+1) x 30(B+7)

If pairs of numbers from each value of A and B are used as products such that;
(7 x 31) x (37 x 61) x (67 x 91) x … and then add 30, the answer is either a prime number or has factors that include a number higher than any of the numbers used above, or has factors from the other equations.

If this exercise is repeated with pairs of numbers from each of the individual equations, then the answer will be a prime or the product of higher prime numbers.