The Density Bands of Goldbach's Conjecture Scatter Plots

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Abstract
Following on from his first paper on Prime Number channels, the author now shows the reasons for the dark banding on these graphs which is based on the number of alignments of these channels per block of 30 numbers.

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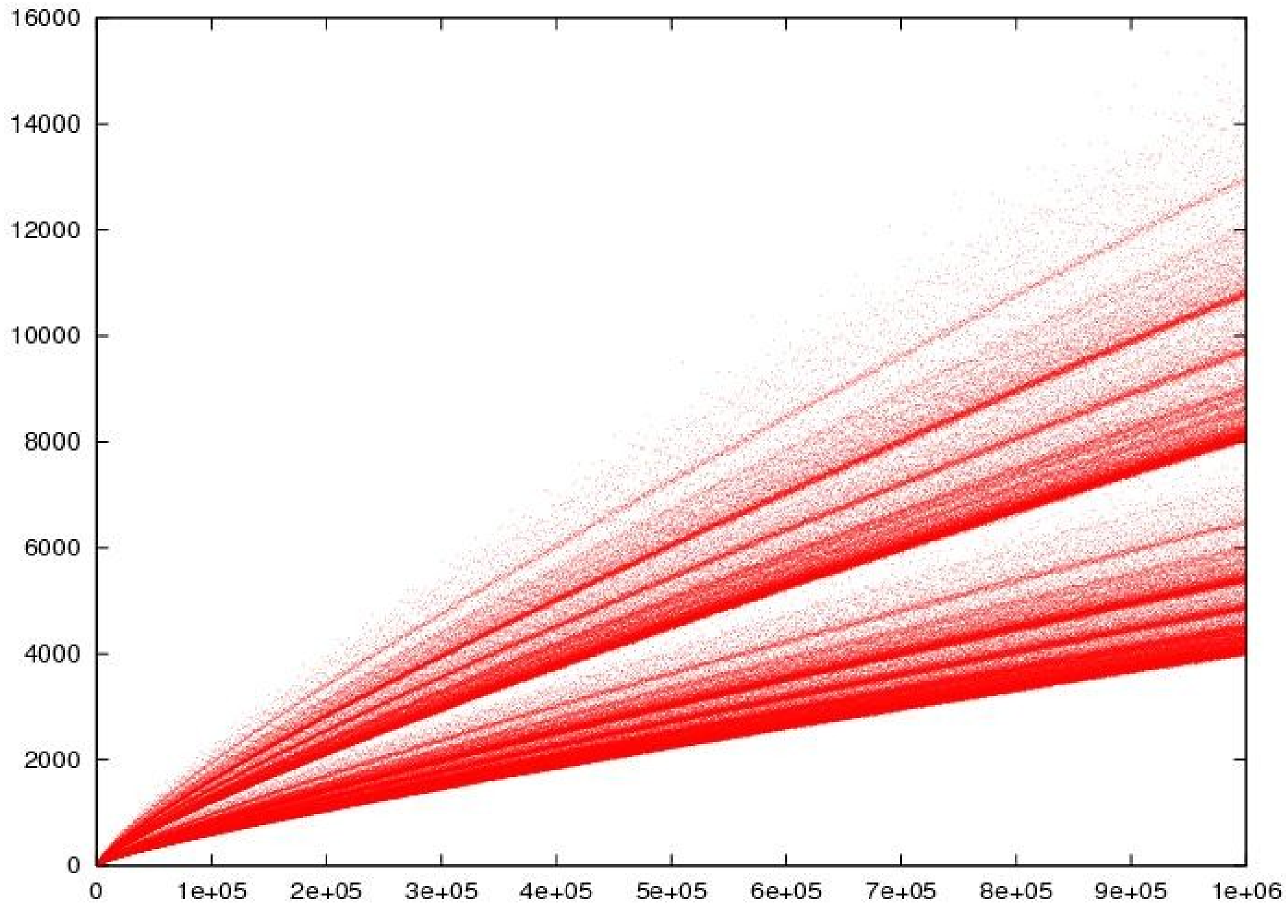
Introduction
The following graph of prime number pairs for even integers up to 1 million has some obvious patterns but no previous simple explanation exists for these to explain them.

Diagram 1 - Pairs of Prime Numbers Scatter Plot



The theoretical maximum number of prime number pairs per block of 30 for alignments of 3, 4, 6 and 8 are as follows;

Table 2 – Maximum Alignments Frequency Per Block of 30

Alignments	**Number of Alignments per Block of 30**
3	8
4	2
6	4
8	1

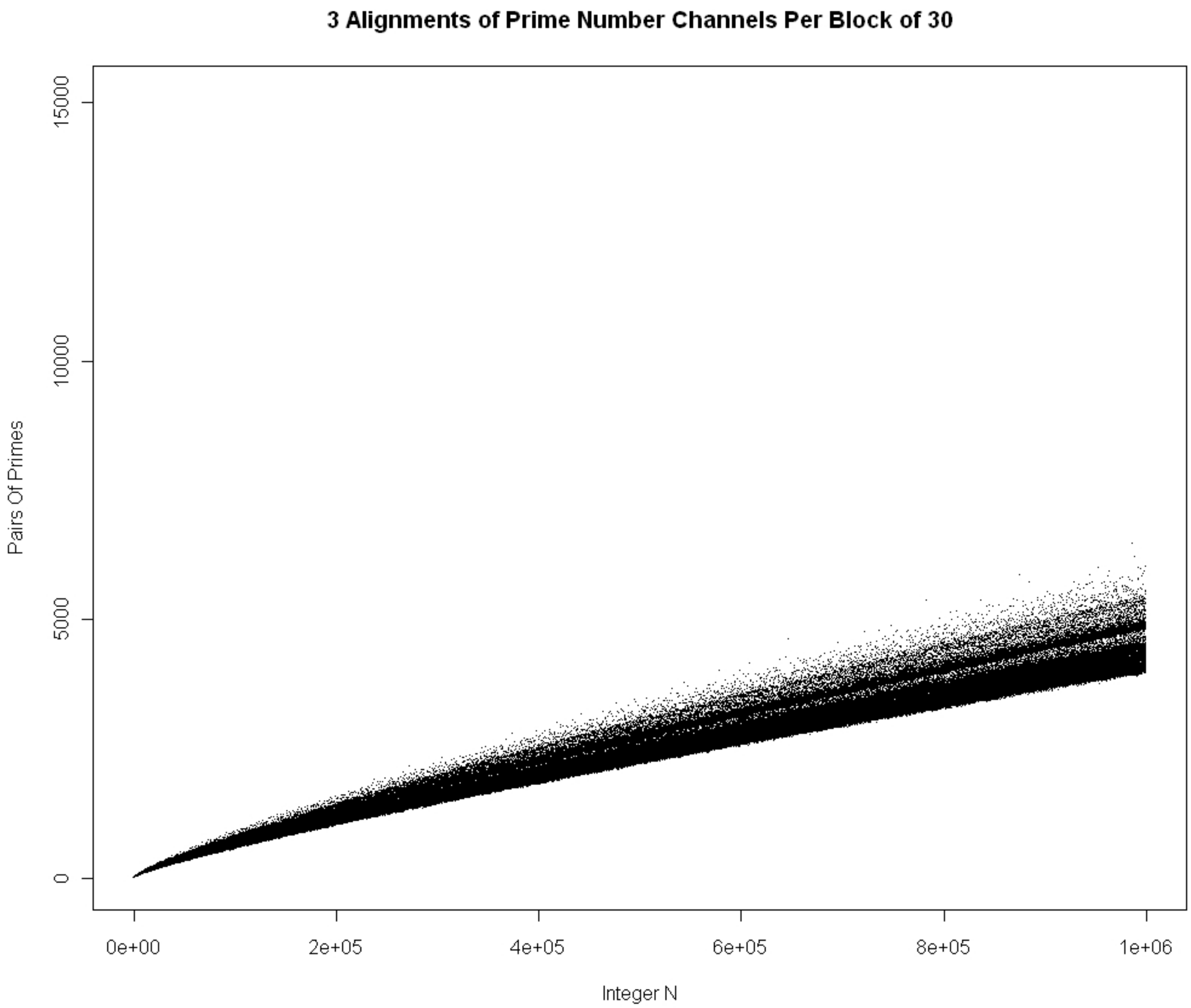
An interesting question is now possible. Do the alignment patterns above manifest themselves in a particular pattern when these are compared against the actual scatter plot of the number of pairs.
There are some very clear lower boundaries with main bands and minor bands. Knowing the alignment densities per block of 30 strongly suggests that if the individual densities are plotted separately, we should get a derived result that is simpler. Also knowing that the densities of some alignments are greater than others should also be obvious if this has any impact on the behaviour of paired primes in this problem.

Note that the lower edges of the bands are more defined than the trailing edges.

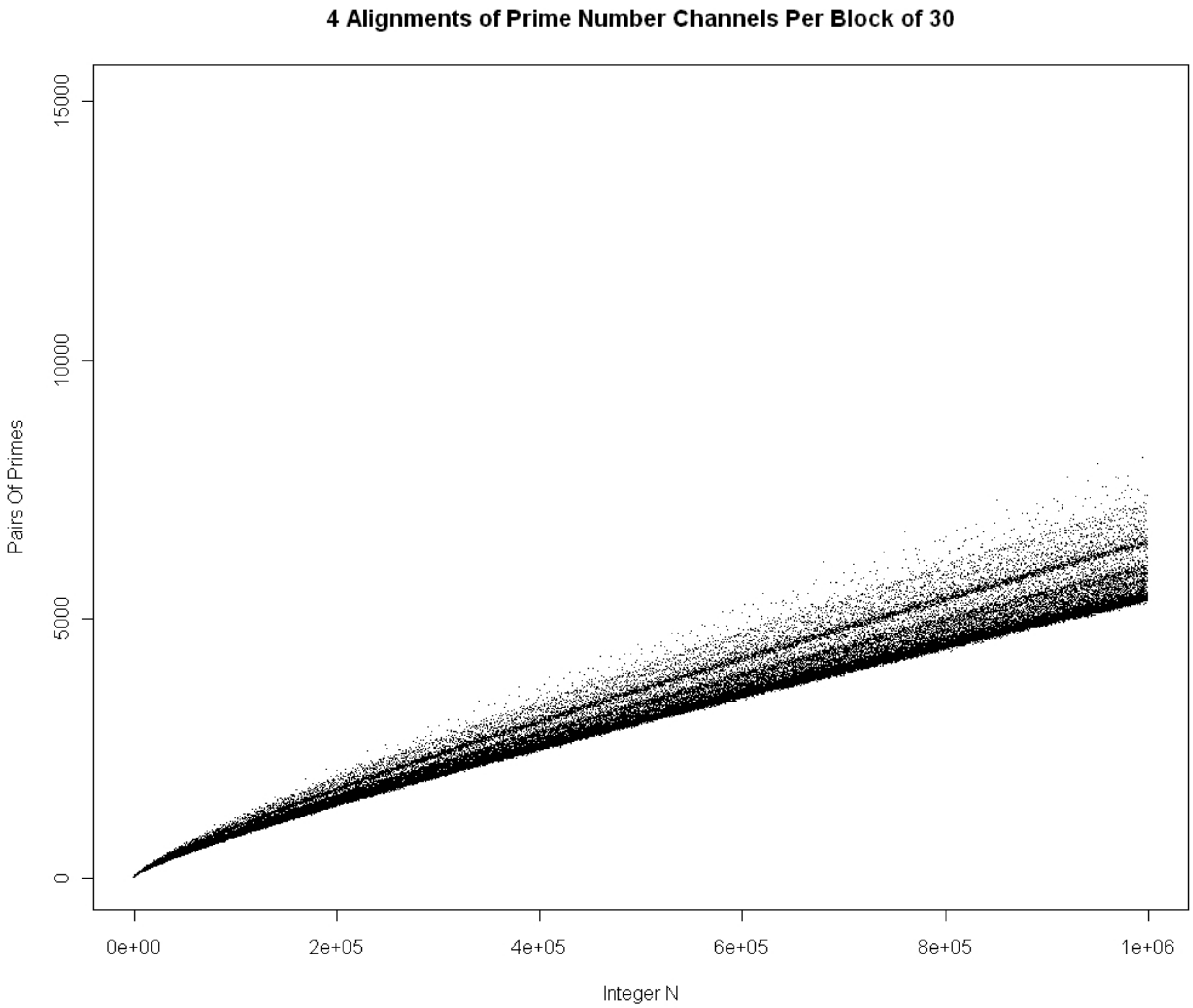
Theoretically the maximum alignment density (8) which happens only once in 15 alignments should generate the most pairs of primes and its scatter graph we could reasonably expect to be about ¼ quarter less points than second maximum alignment of 6. We could also reasonably expect that the third highest density (4) should generate more pairs of primes than the most common density 3.

And lastly because of their relatively densities we will expect that the graphs for each density will have different spreads.

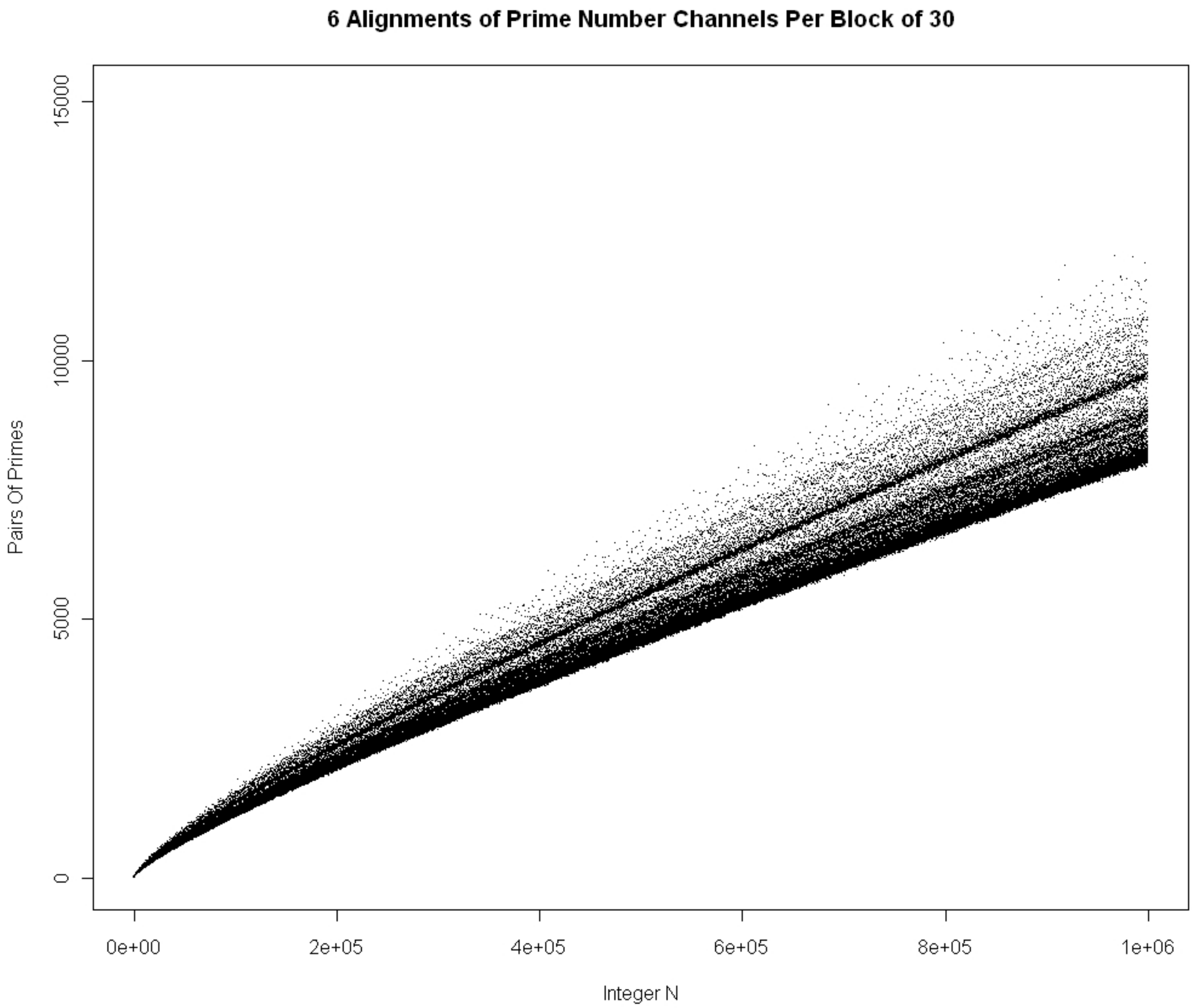
Here are the scatter plots for even numbers for each of the four types of alignments. Each is drawn with the same vertical scale for comparison.



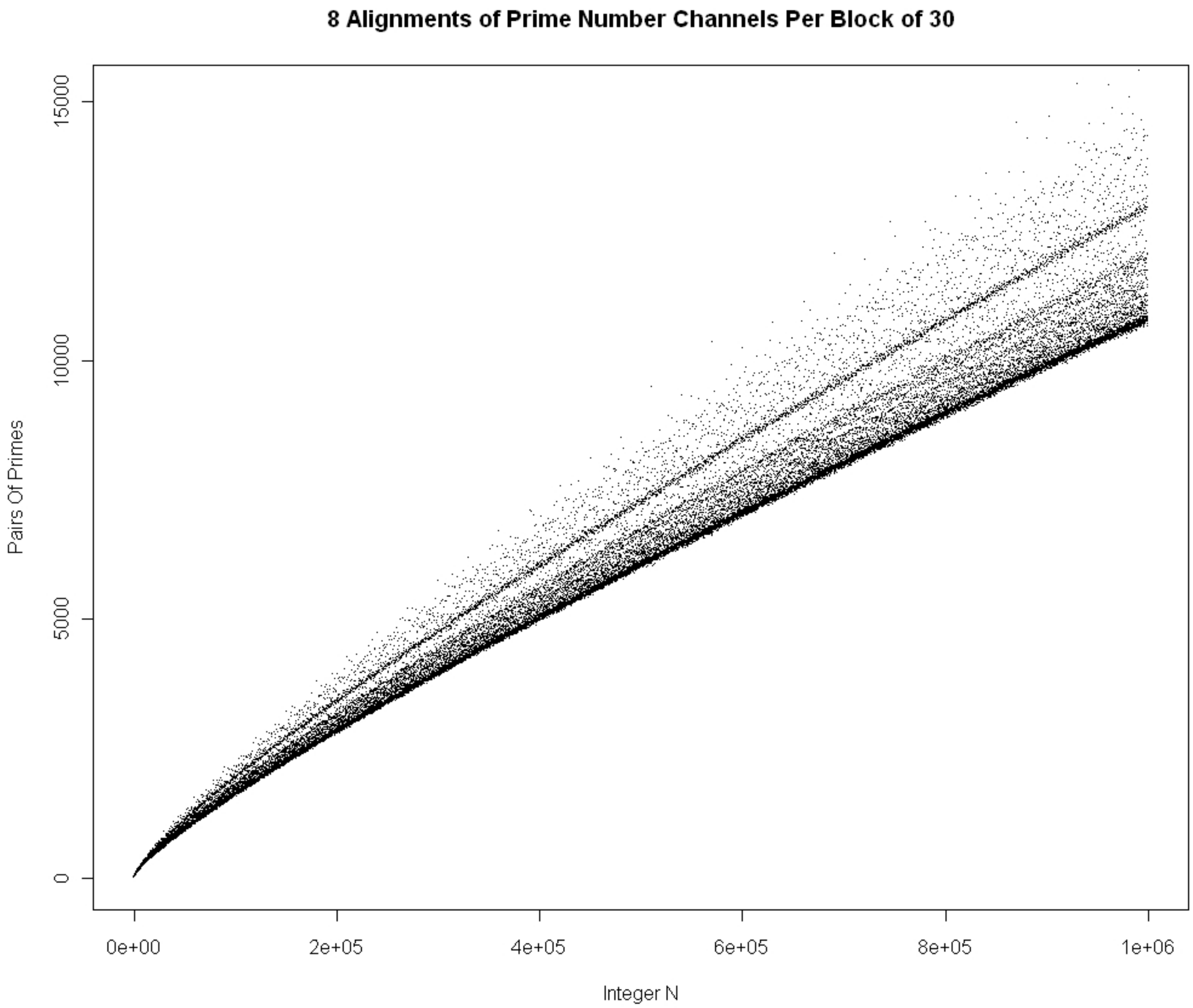
This first scatter plot is the densest for the obvious reason that there are 8 opportunities per block of 30 for a Prime Number Channel to align with another channel. The darkest band shows a concentration of prime number pairs which seems to tail off before a secondary band appears which then also tails off.



This second scatter plot shows the same tailing off with a secondary band also. There is also some evidence visually of some minor bands after the first one.

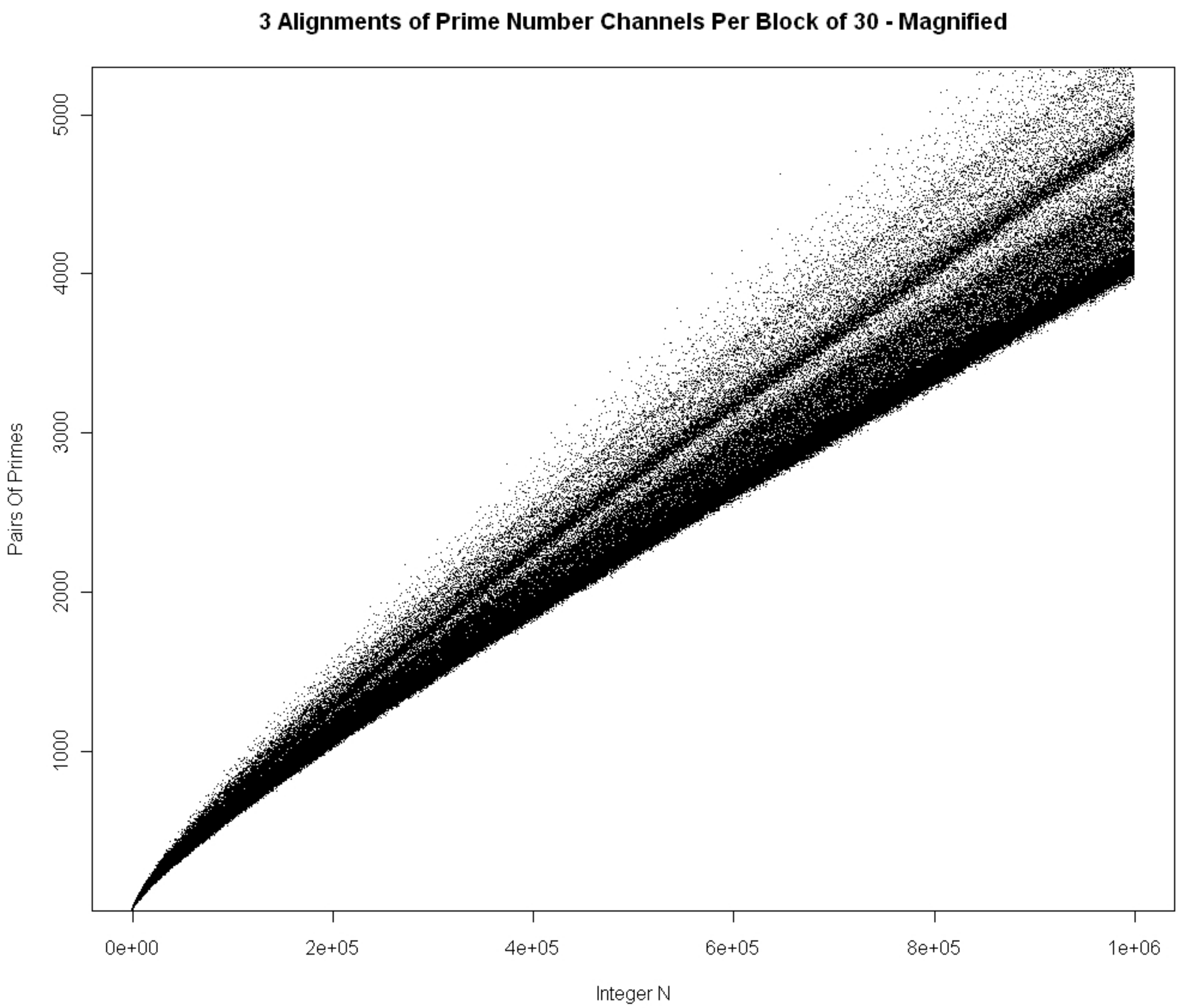


This scatter plot has the same characteristics as the previous two although now the minor bands after the first one are more obvious.



This is the lightest scatter plot due to the 1 in 15 numbers that allow all Prime Number Channels to align with other channels. The same characteristic major and secondary bands appear as well as striations which suggest minor bands again.

The probably reason that the striations do not appear obvious on the first scatter plot is that the density is just too great to observe them. When this scatter plot is expanded then evidence of these striations does start to appear as demonstrated in the following scatter plot.



The next question to resolve is what types of numbers occupy the secondary band as well as the main area of the main band.

All scatter plots share the same characteristics and it is likely that there is something in common with each type of alignment. If the individual scatter plots are stacked on top of each other they form the first scatter plot with only a minimal overlap between 3 and 4, and 6 and 8 with a substantially larger gap between 4 and 6. From 3, to 4, 6 and 8, the vertical span increases in size showing that higher numbers of alignments provide more variability in actual prime number pair matches and that makes sense.

Suggestion: look at data that falls into the darkest band to see if there is something about the positions of those numbers that is relative to multiples of 7 or 7, 11 and 13. Alternatively try different selection criteria for creating subsets of data points (Integer N and number of primes) to see if it is possible to reproduce one of the bands.

The way I built the datasets is relatively easily done. First create a database or array of the first 500,000 prime numbers. Then create a second database or array representing the even numbers up to 1,000,000, each record or element having a field to store the number of prime number pairs and a field to store what alignment type it is.

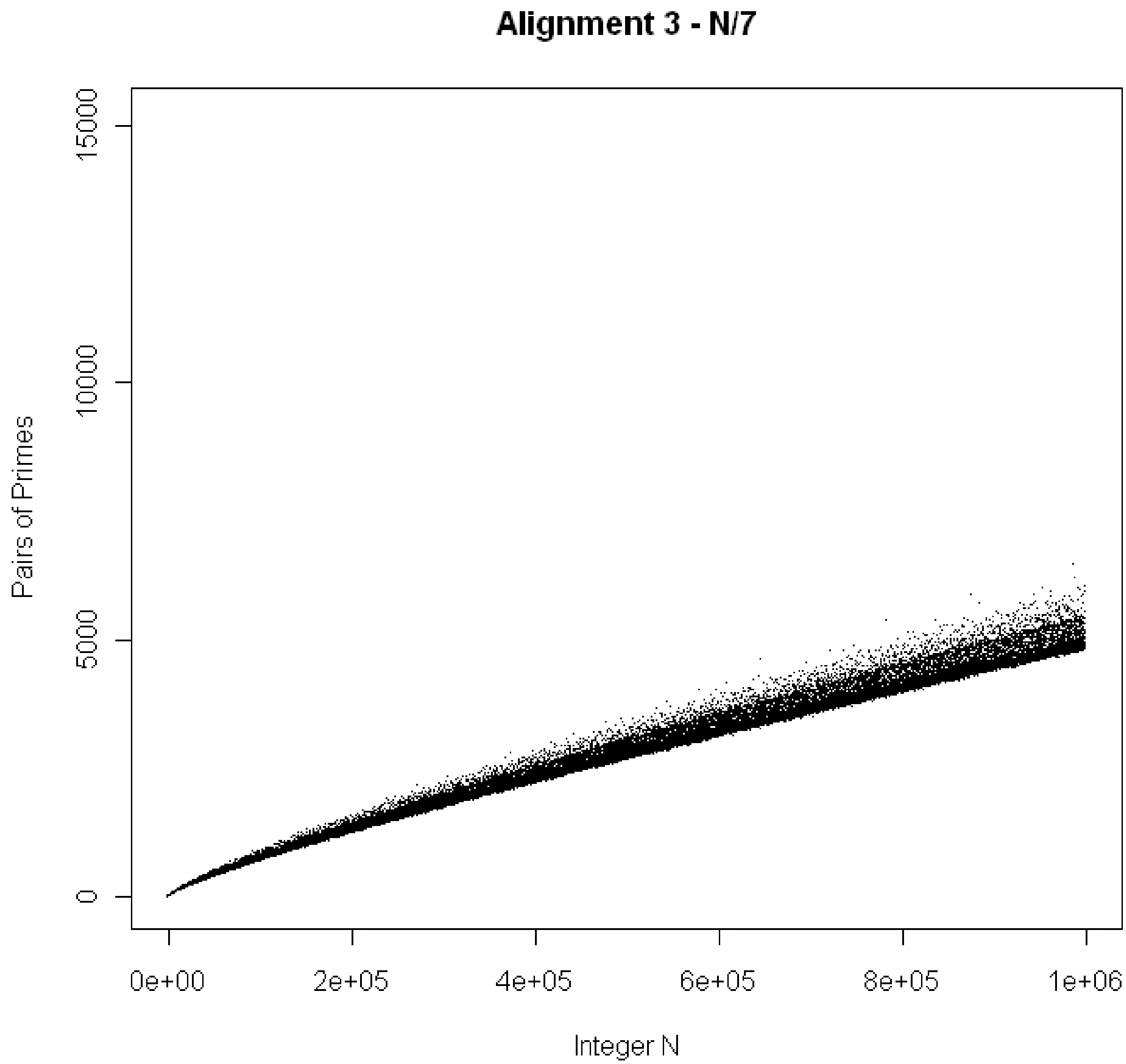
Then create two loops where one starts at the first prime, while the second loop goes through the remainder of the prime numbers, forming different sums which then become an index into the second database or array which has one added to its total of prime pairs seen.

Next set the alignment type as either 3, 4, 6 or 8 for each record. Use the table from my first paper to know which alignment type number to use.

You now have a complete data set from which to pull different combinations of records by alignment type and other conditions.

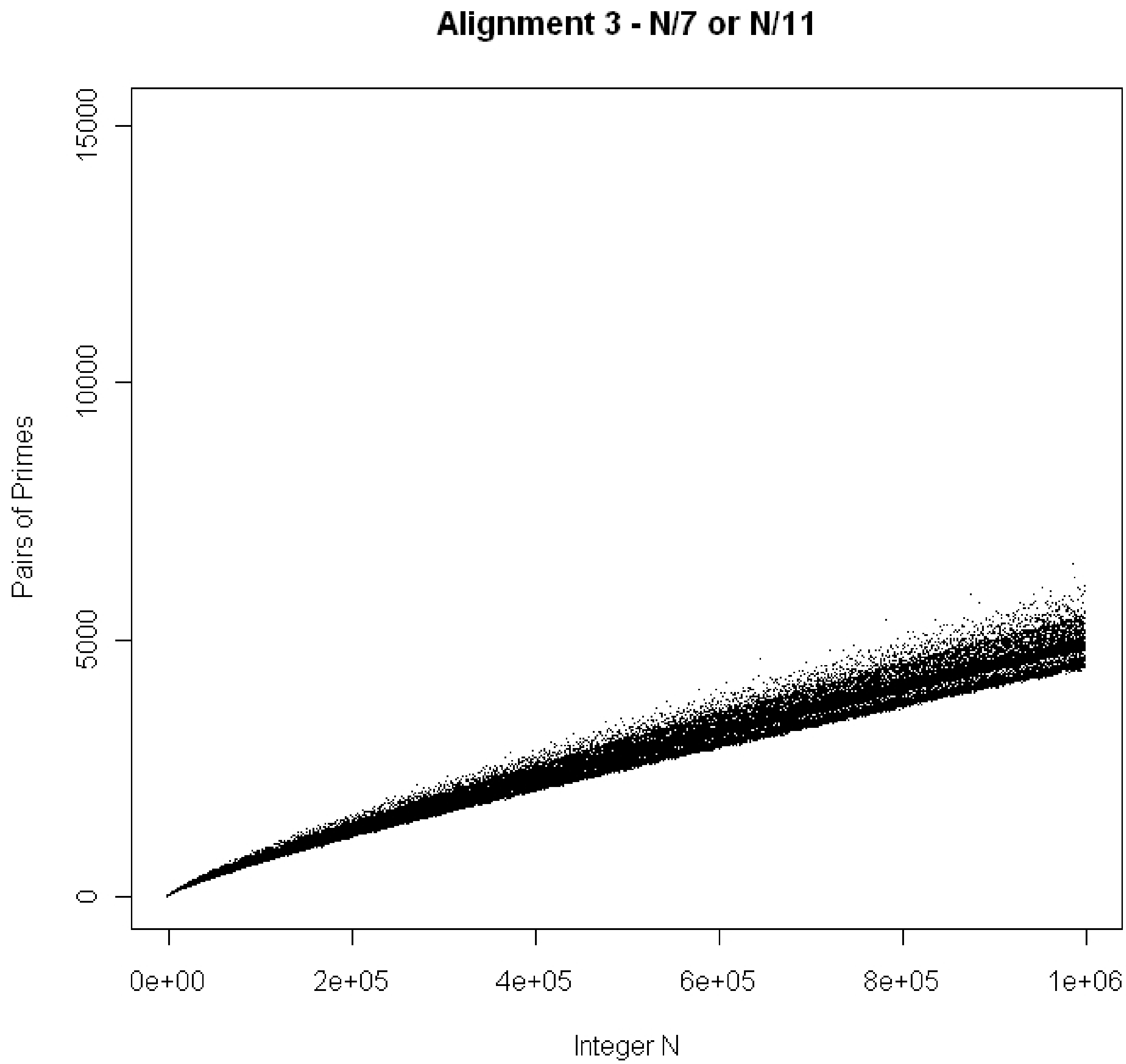
It is clear there are very specific dynamics going on which may help explain some very deep details about prime numbers. Prime number pairs according to the above graphs are concentrated largely around very specific regions with lesser regions attracting lesser quantities.

The next scatter plot is for 3 Alignments per Block of 30 where N is divisible by 7.

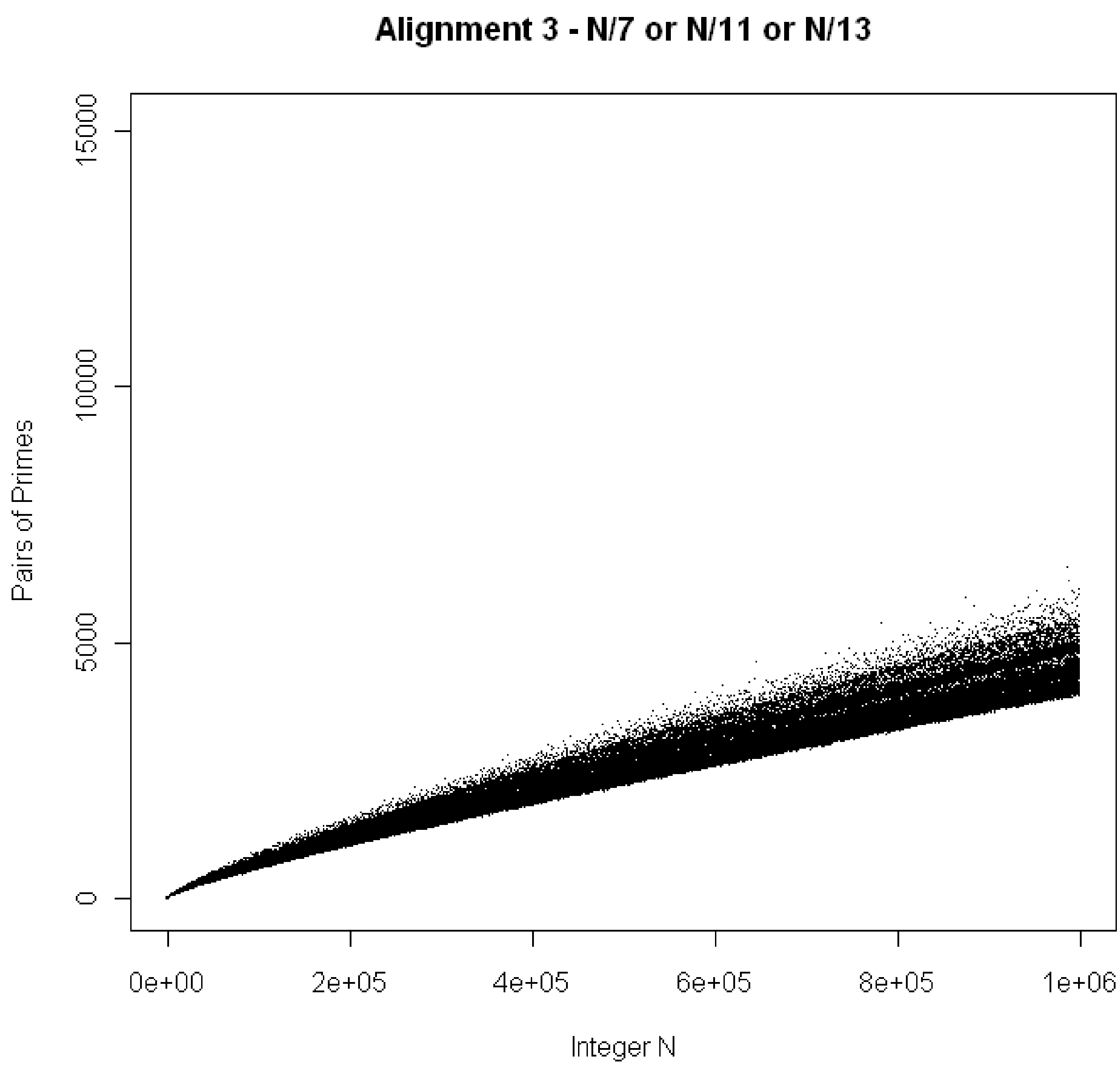


This is very similar to the top part of the first scatter plot for 3 Alignments per Block of 30. This seems reasonable as multiples of 7 are the most common non-prime numbers in the prime number channels.

When N is restricted to multiples of 7 and 11 the scatter plot widens in vertical amplitude as can be seen in the next scatter plot.



If the filter now includes where N is divisible by 7, 11 or 13 – the scatter plot now resembles the full scatter plot.



When looking at the raw unplotted data, some of the points that represent high numbers of prime number pairs are divisible by 11 or 13, but most are divisible by 7. Those that are divisible by 11 and have higher numbers of prime number pairs tend to also be divisible by other low prime numbers such as 7, 13, 17 and 19.

Why does this make such a difference? Take 98 which is divisible by 7. Each pair of odd numbers that are divisible by 7 are also divisible by other factors and its likely that this prevents the same factors restricting the number of pairs of primes. When a member of one pair of odd numbers is divisible by several factors, the next and previous multiples of those factors are even numbers and therefore only able to reduce the numbers of prime number pairs when they are at least double the factor in distance away.

Here are the pairs of odd numbers that are divisible by 7;

7	91 (7 x 13)
21	77 (7 x 11)
35	63 (7 x 3)
49	49

Prime number pairs are as follows;

11	87
19	79
31	67
37	61
41	47

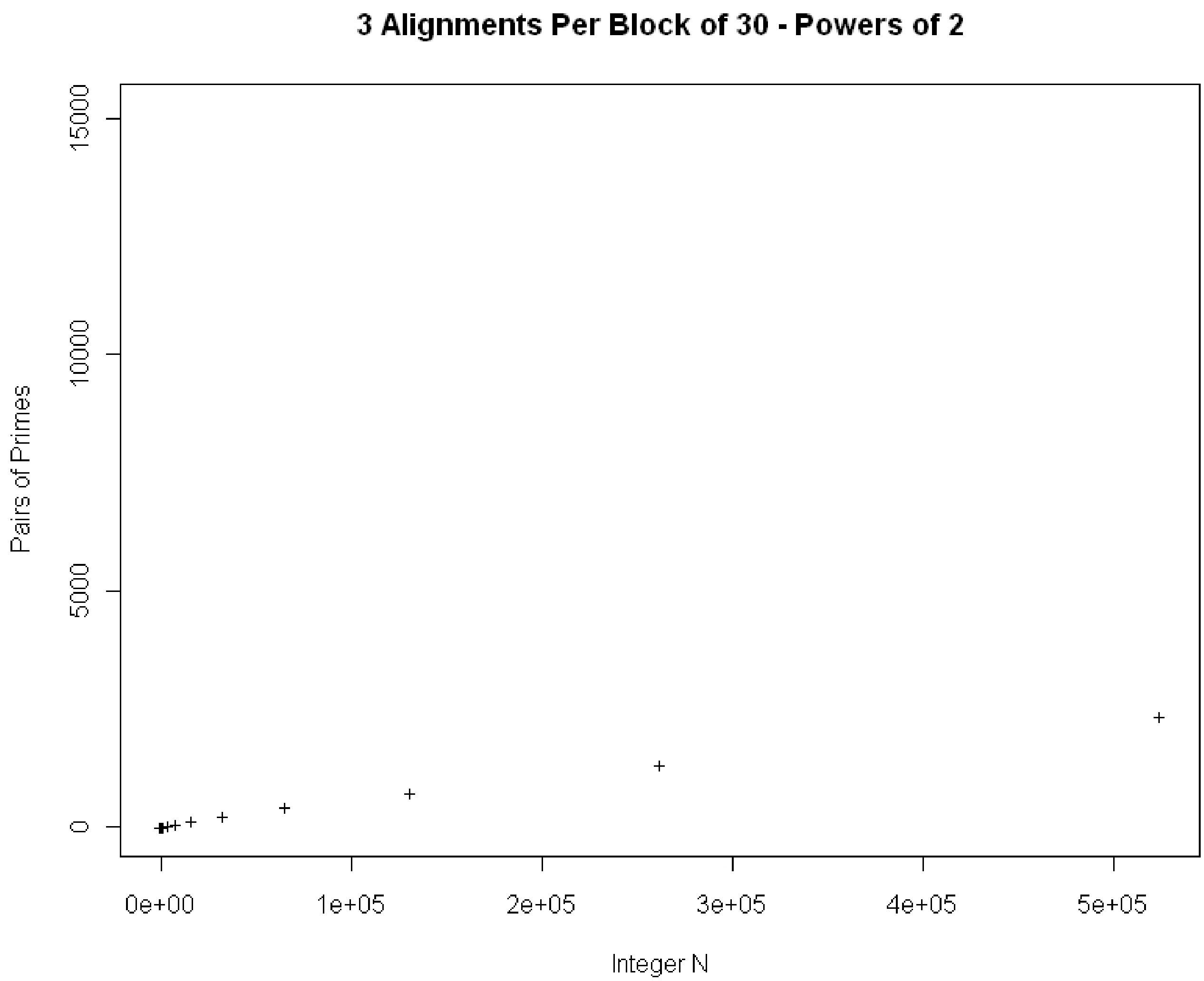
Therefore when N is divisible it itself by many factors, then multiples of those factors take out many other factors and increase the likelihood of more prime number pairs.

Conversely if N has only 2 as a factor, does that mean that it has fewer pairs of primes? Powers of 2 cycle through the same four positions within blocks of multiples of 30. Multiples of 3 and 5 are ignored as they play an increasingly insignificant part in larger value integers.

Power of 2	**Position within Block of 30**	**Alignments Per Block of 30**
32	2	3
64	4	3
128	8	4
256	16	3
512	2	3
1024	4	3
2048	8	4
4096	16	3

Powers of 2 therefore largely fall into the bottom zone of alignments of 3.

The following scatter plot shows the points for this alignment where they exist towards or near the bottom of the normal scatter plot shown previously. It strongly confirms that the number of pairs of primes available for any integer N, depends on the number of lower prime numbers and this excludes 2 itself.



Discussion
The base of each of the four major bands, largely consists of numbers that are multiples of 7, 11 or 13. Because 3 and 5 are irrelevant or trivial primes in respect to this problem, 7 is then the next most common factor of values of N which impacts on the numbers of pairs of prime numbers the most. When N is a multiple of 7, then $1/7^{th}$ of odd number positions have a multiple of 7, and all of these positions pair up with odd number positions also divisible by 7. When N is not divisible by 7, then multiples of 7 cannot pair with multiples of 7. Taking this into account with multiples of 11 and 13, shows why fewer pairs of primes occur when N is divisible by 7,11, or 13.

For different powers of 2, something completely different is happening. Multiples of 3 and 5 cannot pair up to their own multiples and instead pair up with all other available prime numbers. In fact all prime numbers cannot align with multiples of themselves for when N = different powers of 2.

Even though 3 and 5 can align with a prime number, their contribution is still negligible.